

## Workshop Solutions to Section 3.3 (2.6 Ex. 37-56 & page 192,193 Ex.1-36)

<p>1) If <math>f(x) = \begin{cases} 2x + 3; &amp; x \geq -2 \\ 2x + 5; &amp; x &lt; -2 \end{cases}</math> then  <math>\lim_{x \rightarrow (-2)^-} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow (-2)^-} f(x) = \lim_{x \rightarrow (-2)^-} (2x + 5) = 2(-2) + 5 = -4 + 5 = 1</math></p>	<p>2) If <math>f(x) = \begin{cases} 2x + 3; &amp; x \geq -2 \\ 2x + 5; &amp; x &lt; -2 \end{cases}</math> then  <math>\lim_{x \rightarrow (-2)^+} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow (-2)^+} f(x) = \lim_{x \rightarrow (-2)^+} (2x + 3) = 2(-2) + 3 = -4 + 3 = -1</math></p>
<p>3) If <math>f(x) = \begin{cases} 2x + 3; &amp; x \geq -2 \\ 2x + 5; &amp; x &lt; -2 \end{cases}</math> then  <math>\lim_{x \rightarrow -2} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow -2} f(x)</math> does not exist because  <math>\lim_{x \rightarrow (-2)^-} f(x) \neq \lim_{x \rightarrow (-2)^+} f(x)</math></p>	<p>4) If <math>f(x) = \begin{cases} x^2 - 2x + 3; &amp; x \geq 3 \\ x^3 - 3x - 12; &amp; x &lt; 3 \end{cases}</math> then  <math>\lim_{x \rightarrow 3} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x^3 - 3x - 12) = (3)^3 - 3(3) - 12 = 27 - 9 - 12 = 6</math>  <math>\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (x^2 - 2x + 3) = (3)^2 - 2(3) + 3 = 9 - 6 + 3 = 6</math>  <math>\therefore \lim_{x \rightarrow 3} f(x) = 6</math></p>
<p>5) If <math>f(x) = \begin{cases} x^2 - 7x; &amp; x &lt; 1 \\ 5; &amp; 1 \leq x \leq 3 \\ 3x + 1; &amp; x &gt; 3 \end{cases}</math> then  <math>\lim_{x \rightarrow 1^+} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 - 7x) = (1)^2 - 7(1) = 1 - 7 = -6</math></p>	<p>6) If <math>f(x) = \begin{cases} x^2 - 7x; &amp; x &lt; 1 \\ 5; &amp; 1 \leq x \leq 3 \\ 3x + 1; &amp; x &gt; 3 \end{cases}</math> then  <math>\lim_{x \rightarrow 1^+} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (5) = 5</math></p>
<p>7) If <math>f(x) = \begin{cases} x^2 - 7x; &amp; x &lt; 1 \\ 5; &amp; 1 \leq x \leq 3 \\ 3x + 1; &amp; x &gt; 3 \end{cases}</math> then  <math>\lim_{x \rightarrow 3^-} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (5) = 5</math></p>	<p>8) If <math>f(x) = \begin{cases} x^2 - 7x; &amp; x &lt; 1 \\ 5; &amp; 1 \leq x \leq 3 \\ 3x + 1; &amp; x &gt; 3 \end{cases}</math> then  <math>\lim_{x \rightarrow 3^+} f(x) =</math></p> <p><u>Solution:</u>  <math>\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (3x + 1) = 3(3) + 1 = 9 + 1 = 10</math></p>
<p>9) If <math>f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 - 4 &gt; 0 \\ \frac{x^2+x-6}{4-x^2}; &amp; x^2 - 4 &lt; 0 \end{cases}</math> then  <math>\lim_{x \rightarrow 2^+} f(x) =</math></p> <p><u>Solution:</u>  <math>f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 - 4 &gt; 0 \\ \frac{x^2+x-6}{4-x^2}; &amp; x^2 - 4 &lt; 0 \end{cases}</math>  <math>= \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 &gt; 4 \\ \frac{x^2+x-6}{-(x^2-4)}; &amp; x^2 &lt; 4 \end{cases}</math>  <math>= \begin{cases} \frac{(x+3)(x-2)}{(x-2)(x+2)}; &amp;  x  &gt; 4 \\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; &amp;  x  &lt; 4 \end{cases}</math>  <math>= \begin{cases} \frac{x+3}{x+2}; &amp; x &gt; 2 \text{ or } x &lt; -2 \\ -\frac{x+3}{x+2}; &amp; -2 &lt; x &lt; 2 \end{cases}</math> then  <math>\therefore \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \left( \frac{x+3}{x+2} \right) = \frac{(2)+3}{(2)+2} = \frac{5}{4}</math></p>	<p>10) If <math>f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 - 4 &gt; 0 \\ \frac{x^2+x-6}{4-x^2}; &amp; x^2 - 4 &lt; 0 \end{cases}</math> then  <math>\lim_{x \rightarrow 2^-} f(x) =</math></p> <p><u>Solution:</u>  <math>f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 - 4 &gt; 0 \\ \frac{x^2+x-6}{4-x^2}; &amp; x^2 - 4 &lt; 0 \end{cases}</math>  <math>= \begin{cases} \frac{x^2+x-6}{x^2-4}; &amp; x^2 &gt; 4 \\ \frac{x^2+x-6}{-(x^2-4)}; &amp; x^2 &lt; 4 \end{cases}</math>  <math>= \begin{cases} \frac{(x+3)(x-2)}{(x-2)(x+2)}; &amp;  x  &gt; 4 \\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; &amp;  x  &lt; 4 \end{cases}</math>  <math>= \begin{cases} \frac{x+3}{x+2}; &amp; x &gt; 2 \text{ or } x &lt; -2 \\ -\frac{x+3}{x+2}; &amp; -2 &lt; x &lt; 2 \end{cases}</math> then  <math>\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \left( -\frac{x+3}{x+2} \right) = -\frac{(2)+3}{(2)+2} = -\frac{5}{4}</math></p>

11)

$$\lim_{x \rightarrow a^-} \frac{|x-a|}{x-a} =$$

Solution:

$$f(x) = \frac{|x-a|}{x-a} = \begin{cases} \frac{x-a}{x-a} & ; x-a > 0 \\ \frac{-(x-a)}{x-a} & ; x-a < 0 \end{cases} = \begin{cases} 1; & x > a \\ -1; & x < a \end{cases}$$

$$\therefore \lim_{x \rightarrow a^-} \frac{|x-a|}{x-a} = \lim_{x \rightarrow a^-} \frac{-(x-a)}{x-a} = \lim_{x \rightarrow a^-} (-1) = -1$$

12)

$$\lim_{x \rightarrow a^+} \frac{|x-a|}{x-a} =$$

Solution:

$$f(x) = \frac{|x-a|}{x-a} = \begin{cases} \frac{x-a}{x-a} & ; x-a > 0 \\ \frac{-(x-a)}{x-a} & ; x-a < 0 \end{cases} = \begin{cases} 1; & x > a \\ -1; & x < a \end{cases}$$

$$\therefore \lim_{x \rightarrow a^+} \frac{|x-a|}{x-a} = \lim_{x \rightarrow a^+} \frac{(x-a)}{x-a} = \lim_{x \rightarrow a^+} (1) = 1$$

13)

$$\lim_{x \rightarrow a} \frac{|x-a|}{x-a} =$$

Solution:

$\lim_{x \rightarrow a} \frac{|x-a|}{x-a}$  does not exist because

$$\lim_{x \rightarrow a^-} \frac{|x-a|}{x-a} \neq \lim_{x \rightarrow a^+} \frac{|x-a|}{x-a}$$

It is clearly obvious from questions (11) and (12) above.

14)

$$\lim_{x \rightarrow a^+} \frac{|a-x|}{x-a} =$$

Solution:

$$f(x) = \frac{|a-x|}{x-a} = \begin{cases} \frac{a-x}{x-a} & ; a-x > 0 \\ \frac{-(a-x)}{x-a} & ; a-x < 0 \end{cases}$$

$$= \begin{cases} \frac{-(x-a)}{x-a} & ; a > x \\ \frac{(x-a)}{x-a} & ; a < x \end{cases} = \begin{cases} -1; & x < a \\ 1; & x > a \end{cases}$$

$$\therefore \lim_{x \rightarrow a^+} \frac{|a-x|}{x-a} = \lim_{x \rightarrow a^+} (1) = 1$$

15)

$$\lim_{x \rightarrow a^-} \frac{|a-x|}{x-a} =$$

Solution:

$$f(x) = \frac{|a-x|}{x-a} = \begin{cases} \frac{a-x}{x-a} & ; a-x > 0 \\ \frac{-(a-x)}{x-a} & ; a-x < 0 \end{cases}$$

$$= \begin{cases} \frac{-(x-a)}{x-a} & ; a > x \\ \frac{(x-a)}{x-a} & ; a < x \end{cases} = \begin{cases} -1; & x < a \\ 1; & x > a \end{cases}$$

$$\therefore \lim_{x \rightarrow a^-} \frac{|a-x|}{x-a} = \lim_{x \rightarrow a^-} (-1) = -1$$

16)

$$\lim_{x \rightarrow a} \frac{|a-x|}{x-a} =$$

Solution:

$\lim_{x \rightarrow a} \frac{|a-x|}{x-a}$  does not exist because

$$\lim_{x \rightarrow a^-} \frac{|a-x|}{x-a} \neq \lim_{x \rightarrow a^+} \frac{|a-x|}{x-a}$$

It is clearly obvious from questions (14) and (15) above.

17)

$$\lim_{x \rightarrow (-a)^-} \frac{|x+a|}{x+a} =$$

Solution:

$$f(x) = \frac{|x+a|}{x+a} = \begin{cases} \frac{x+a}{x+a} & ; x+a > 0 \\ \frac{-(x+a)}{x+a} & ; x+a < 0 \end{cases} = \begin{cases} 1; & x > -a \\ -1; & x < -a \end{cases}$$

$$\therefore \lim_{x \rightarrow (-a)^-} \frac{|x+a|}{x+a} = \lim_{x \rightarrow (-a)^-} (-1) = -1$$

18)

$$\lim_{x \rightarrow (-a)^+} \frac{|x+a|}{x+a} =$$

Solution:

$$f(x) = \frac{|x+a|}{x+a} = \begin{cases} \frac{x+a}{x+a} & ; x+a > 0 \\ \frac{-(x+a)}{x+a} & ; x+a < 0 \end{cases} = \begin{cases} 1; & x > -a \\ -1; & x < -a \end{cases}$$

$$\therefore \lim_{x \rightarrow (-a)^+} \frac{|x+a|}{x+a} = \lim_{x \rightarrow (-a)^+} (1) = 1$$

19)

$$\lim_{x \rightarrow -a} \frac{|x+a|}{x+a} =$$

Solution:

$\lim_{x \rightarrow -a} \frac{|x+a|}{x+a}$  does not exist because

$$\lim_{x \rightarrow (-a)^-} \frac{|x+a|}{x+a} \neq \lim_{x \rightarrow (-a)^+} \frac{|x+a|}{x+a}$$

It is clearly obvious from questions (17) and (18) above.

20)

$$\lim_{x \rightarrow 0^+} \frac{2x - |x|}{x^2 + |x|} =$$

Solution:

$$\begin{aligned} f(x) = \frac{2x - |x|}{x^2 + |x|} &= \begin{cases} \frac{2x - (x)}{x^2 + (x)} & ; x > 0 \\ \frac{2x - (-x)}{x^2 + (-x)} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{2x - x}{x^2 + x} & ; x > 0 \\ \frac{2x + x}{x^2 - x} & ; x < 0 \end{cases} = \begin{cases} \frac{x}{x^2 + x} & ; x > 0 \\ \frac{3x}{x^2 - x} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{x(x+1)}{3x} & ; x > 0 \\ \frac{3x}{x(x-1)} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{1}{x+1} & ; x > 0 \\ \frac{3}{x-1} & ; x < 0 \end{cases} \\ \therefore \lim_{x \rightarrow 0^+} \frac{2x - |x|}{x^2 + |x|} &= \lim_{x \rightarrow 0^+} \frac{1}{x+1} = \frac{1}{0+1} = 1 \end{aligned}$$

21)

$$\lim_{x \rightarrow 0^-} \frac{2x - |x|}{x^2 + |x|} =$$

Solution:

$$\begin{aligned} f(x) = \frac{2x - |x|}{x^2 + |x|} &= \begin{cases} \frac{2x - (x)}{x^2 + (x)} & ; x > 0 \\ \frac{2x - (-x)}{x^2 + (-x)} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{2x - x}{x^2 + x} & ; x > 0 \\ \frac{2x + x}{x^2 - x} & ; x < 0 \end{cases} = \begin{cases} \frac{x}{x^2 + x} & ; x > 0 \\ \frac{3x}{x^2 - x} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{x(x+1)}{3x} & ; x > 0 \\ \frac{3x}{x(x-1)} & ; x < 0 \end{cases} \\ &= \begin{cases} \frac{1}{x+1} & ; x > 0 \\ \frac{3}{x-1} & ; x < 0 \end{cases} \\ \therefore \lim_{x \rightarrow 0^-} \frac{2x - |x|}{x^2 + |x|} &= \lim_{x \rightarrow 0^-} \frac{3}{x-1} = \frac{3}{0-1} = -3 \end{aligned}$$

22)

$$\lim_{x \rightarrow 0} \frac{2x - |x|}{x^2 + |x|} =$$

Solution:

$\lim_{x \rightarrow 0} \frac{2x - |x|}{x^2 + |x|}$  does not exist because

$$\lim_{x \rightarrow 0^-} \frac{2x - |x|}{x^2 + |x|} \neq \lim_{x \rightarrow 0^+} \frac{2x - |x|}{x^2 + |x|}$$

It is clearly obvious from questions (20) and (21) above.

23)

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x - \sin x)(\cos x + \sin x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\cos x + \sin x} = \frac{1}{\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)} \\ &= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2}}{2} \end{aligned}$$

24)

$$\lim_{x \rightarrow 0} \frac{\cos^2 x + 2 \cos x - 3}{2 \cos^2 x - \cos x - 1} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\cos^2 x + 2 \cos x - 3}{2 \cos^2 x - \cos x - 1} &= \lim_{x \rightarrow 0} \frac{(\cos x + 3)(\cos x - 1)}{(2 \cos x + 1)(\cos x - 1)} \\ &= \lim_{x \rightarrow 0} \frac{\cos x + 3}{2 \cos x + 1} = \frac{\cos(0) + 3}{2 \cos(0) + 1} \\ &= \frac{1 + 3}{2(1) + 1} = \frac{4}{3} \end{aligned}$$

25)

$$\lim_{x \rightarrow 0} (\sin^2 x + 3 \tan x - 4) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} (\sin^2 x + 3 \tan x - 4) &= \sin^2(0) + 3 \tan(0) - 4 \\ &= 0 + 3(0) - 4 = -4 \end{aligned}$$

26) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\sin(nx)}{mx} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin(nx)}{mx} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} = \frac{n}{m} (1) = \frac{n}{m}$$

27) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\tan(nx)}{mx} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\tan(nx)}{mx} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} = \frac{n}{m} (1) = \frac{n}{m}$$

28) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{nx}{\sin(mx)} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{nx}{\sin(mx)} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} = \frac{n}{m} (1) = \frac{n}{m}$$

29) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{nx}{\tan(mx)} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{nx}{\tan(mx)} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} = \frac{n}{m} (1) = \frac{n}{m}$$

30) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\sin(nx)}{\sin(mx)} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(nx)}{\sin(mx)} &= \frac{n}{m} \left( \lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} \right) \left( \lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$$

31) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\sin(nx)}{\tan(mx)} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(nx)}{\tan(mx)} &= \frac{n}{m} \left( \lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} \right) \left( \lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$$

32) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\tan(nx)}{\tan(mx)} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan(nx)}{\tan(mx)} &= \frac{n}{m} \left( \lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} \right) \left( \lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$$

33) If  $m \neq 0$ , then

$$\lim_{x \rightarrow 0} \frac{\tan(nx)}{\sin(mx)} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan(nx)}{\sin(mx)} &= \frac{n}{m} \left( \lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} \right) \left( \lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$$

34)

$$\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x} = 1$$

35)

$$\lim_{x \rightarrow 0} \frac{\sin(\sin(2x))}{\sin(2x)} =$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin(\sin(2x))}{\sin(2x)} = 1$$

36)

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} &= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^2 \\ &= 2 \left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2 = 2(1)^2 = 2 \end{aligned}$$

37)

$$\lim_{x \rightarrow \infty} \sqrt{\frac{1}{x^2} - \frac{3}{x} + 4} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \sqrt{\frac{1}{x^2} - \frac{3}{x} + 4} &= \sqrt{\lim_{x \rightarrow \infty} \left( \frac{1}{x^2} - \frac{3}{x} + 4 \right)} = \sqrt{0 - 0 + 4} \\ &= 2 \end{aligned}$$

38)

$$\lim_{x \rightarrow \infty} \left( \frac{1}{x^{2/5}} + 2 \right) =$$

Solution:

$$\lim_{x \rightarrow \infty} \left( \frac{1}{x^{2/5}} + 2 \right) = 0 + 2 = 2$$

39)

$$\lim_{x \rightarrow \infty} \frac{3x + 15}{9x^2 + 4x - 13} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{0 + 0}{9 + 0 + 0} = 0 \end{aligned}$$

40)

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x^2}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{3 - \frac{8}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{3 - 0 + 0}{9 + 0 + 0} = \frac{1}{3} \end{aligned}$$

41)

$$\lim_{x \rightarrow -\infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x^2}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-3 + \frac{8}{x} - \frac{15}{x^2}}{-9 - \frac{4}{x} + \frac{13}{x^2}} = \frac{-3 + 0 - 0}{-9 - 0 + 0} = \frac{1}{3} \end{aligned}$$

42)

$$\lim_{x \rightarrow \infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x^5}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{3x^3 - \frac{8}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{3(\infty) - 0 + 0}{9 + 0 + 0} = \infty \end{aligned}$$

43)

$$\lim_{x \rightarrow -\infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x^5}{-x^2} - \frac{8x}{-x^2} + \frac{15}{-x^2}}{\frac{9x^2}{-x^2} + \frac{4x}{-x^2} - \frac{13}{-x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-3x^3 + \frac{8}{x} - \frac{15}{x^2}}{-9 - \frac{4}{x} + \frac{13}{x^2}} = \frac{-3(-\infty) + 0 - 0}{-9 - 0 + 0} = -\infty \end{aligned}$$

44)

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 7} - x) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 7} - x) &= \lim_{x \rightarrow \infty} \left[ (\sqrt{x^2 - 3x + 7} - x) \times \frac{(\sqrt{x^2 - 3x + 7} + x)}{(\sqrt{x^2 - 3x + 7} + x)} \right] \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 - 3x + 7) - x^2}{(\sqrt{x^2 - 3x + 7} + x)} = \lim_{x \rightarrow \infty} \frac{-3x + 7}{\sqrt{x^2 - 3x + 7} + x} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{-3x}{x} + \frac{7}{x}}{\frac{\sqrt{x^2 - 3x + 7}}{x} + \frac{x}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{-3 + \frac{7}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{7}{x^2}} + 1} \\ &= \lim_{x \rightarrow \infty} \frac{-3 + \frac{7}{x}}{\sqrt{1 - \frac{3}{x} + \frac{7}{x^2}} + 1} \\ &= \frac{-3 + 0}{\sqrt{1 - 0 + 0} + 1} = \frac{-3}{1 + 1} = -\frac{3}{2} \end{aligned}$$

45)

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) &= \lim_{x \rightarrow \infty} \left[ (\sqrt{x^2 + x} - x) \times \frac{(\sqrt{x^2 + x} + x)}{(\sqrt{x^2 + x} + x)} \right] \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + x) - x^2}{(\sqrt{x^2 + x} + x)} \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + x} + x} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2 + x}}{x} + \frac{x}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + 1} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{\sqrt{1 + 0} + 1} = \frac{1}{1 + 1} \\ &= \frac{1}{2} \end{aligned}$$

46)

$$\lim_{x \rightarrow \infty} (x^2 - 5x + 4) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} (x^2 - 5x + 4) &= \lim_{x \rightarrow \infty} x^2 \left( \frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{4}{x^2} \right) \\ &= \lim_{x \rightarrow \infty} x^2 \left( 1 - \frac{5}{x} + \frac{4}{x^2} \right) = (\infty)^2 (1 - 0 + 0) = \infty \end{aligned}$$

**OR**

$$\lim_{x \rightarrow \infty} (x^2 - 5x + 4) = \lim_{x \rightarrow \infty} (x^2) = \infty$$

47)

$$\lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) &= \lim_{x \rightarrow -\infty} x^4 \left( \frac{x^4}{x^4} - \frac{2x^3}{x^4} + \frac{9}{x^4} \right) \\ &= \lim_{x \rightarrow -\infty} x^4 \left( 1 - \frac{2}{x} + \frac{9}{x^4} \right) = (-\infty)^4 (1 - 0 + 0) = \infty \end{aligned}$$

**OR**

$$\lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) = \lim_{x \rightarrow -\infty} (x^4) = \infty$$

48)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{3x^2 - 8}}{-x} + \frac{2}{-x}}{\frac{x}{-x} + \frac{5}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{3x^2 - 8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{3x^2}{x^2} - \frac{8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{3 - \frac{8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} = \frac{\sqrt{3 - 0} - 0}{-1 - 0} = -\sqrt{3} \end{aligned}$$

49)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{3x^2 - 8}}{x} + \frac{2}{x}}{\frac{x}{x} + \frac{5}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{3x^2 - 8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{3x^2}{x^2} - \frac{8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{3 - \frac{8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} = \frac{\sqrt{3 - 0} + 0}{1 + 0} = \sqrt{3} \end{aligned}$$

50) The horizontal asymptotes of

$$f(x) = \frac{\sqrt{3x^2 - 8} + 2}{x + 5}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5}$$

It is clear from the previous questions (48) and (49) that

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \sqrt{3}$$

and

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = -\sqrt{3}$$

Thus, the horizontal asymptotes are

$$y = \pm\sqrt{3}$$

51) The horizontal asymptote of

$$f(x) = \frac{1 - x}{2x + 1}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{1 - x}{2x + 1}$$

$$\lim_{x \rightarrow \infty} \frac{1 - x}{2x + 1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{x}{x}}{\frac{2x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 1}{2 + \frac{1}{x}} = \frac{0 - 1}{2 + 0} = -\frac{1}{2}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{1 - x}{2x + 1} &= \lim_{x \rightarrow -\infty} \frac{\frac{1}{-x} - \frac{-x}{-x}}{\frac{2x}{-x} + \frac{1}{-x}} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{-x} + 1}{-2 - \frac{1}{x}} = \frac{0 + 1}{-2 - 0} \\ &= -\frac{1}{2} \end{aligned}$$

Thus, the horizontal asymptote is

$$y = -\frac{1}{2}$$

52) The horizontal asymptote of

$$f(x) = \frac{7x^2 + 5}{3x^2 + 2}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{7x^2 + 5}{3x^2 + 2}$$

$$\lim_{x \rightarrow \infty} \frac{7x^2 + 5}{3x^2 + 2} = \lim_{x \rightarrow \infty} \frac{\frac{7x^2}{x^2} + \frac{5}{x^2}}{\frac{3x^2}{x^2} + \frac{2}{x^2}} = \lim_{x \rightarrow \infty} \frac{7 + \frac{5}{x^2}}{3 + \frac{2}{x^2}} = \frac{7 + 0}{3 + 0} = \frac{7}{3}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{7x^2 + 5}{3x^2 + 2} &= \lim_{x \rightarrow -\infty} \frac{\frac{7x^2}{-x^2} + \frac{5}{-x^2}}{\frac{3x^2}{-x^2} + \frac{2}{-x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-7 - \frac{5}{x^2}}{-3 - \frac{2}{x^2}} = \frac{-7 - 0}{-3 - 0} = \frac{7}{3} \end{aligned}$$

Thus, the horizontal asymptote is

$$y = \frac{7}{3}$$

53) The horizontal asymptote of

$$f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} &= \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x - 3}}{\frac{2x}{x} + \frac{7}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2 + 2x - 3}{x^2}}}{2 + \frac{7}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + \frac{2x}{x^2} - \frac{3}{x^2}}}{2 + \frac{7}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{x} - \frac{3}{x^2}}}{2 + \frac{7}{x}} = \frac{\sqrt{1 + 0 - 0}}{2 + 0} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x - 3}}{\frac{2x}{-x} + \frac{7}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{x^2 + 2x - 3}{x^2}}}{-2 - \frac{7}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + \frac{2x}{x^2} - \frac{3}{x^2}}}{-2 - \frac{7}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{1 + \frac{2}{x} - \frac{3}{x^2}}}{-2 - \frac{7}{x}} = \frac{\sqrt{1 + 0 - 0}}{-2 - 0} = -\frac{1}{2} \end{aligned}$$

Thus, the horizontal asymptotes are

$$y = \pm \frac{1}{2}$$

54) The horizontal asymptote of

$$f(x) = \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1} &= \lim_{x \rightarrow \infty} \frac{\sqrt{2x - 3}}{\frac{2x^2}{x^2} + \frac{7x}{x^2} - \frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2x - 3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2x}{x^4} - \frac{3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2}{x^3} - \frac{3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} = \frac{\sqrt{0 - 0}}{2 + 0 - 0} = \frac{0}{2} = 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{2x - 3}}{\frac{2x^2}{-x^2} + \frac{7x}{-x^2} - \frac{1}{-x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2x - 3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2x}{x^4} - \frac{3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2}{x^3} - \frac{3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} = \frac{\sqrt{0 - 0}}{-2 - 0 + 0} = \frac{0}{-2} = 0 \end{aligned}$$

Thus, the horizontal asymptote is

$$y = 0$$

55)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{4x^2 - 8}}{-x} + \frac{3}{-x}}{\frac{x}{-x} + \frac{1}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{4x^2 - 8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - \frac{8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{4 - \frac{8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} = \frac{\sqrt{4 - 0} - 0}{-1 - 0} = -2 \end{aligned}$$

56)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{4x^2 - 8}}{x} + \frac{3}{x}}{\frac{x}{x} + \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{4x^2 - 8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - \frac{8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} = \frac{\sqrt{4 - 0} + 0}{1 + 0} = 2 \end{aligned}$$